An explanation of the "negative neutrino mass squared" anomaly in tritium β-decay based on a theory of mass

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A proposed solution of the anomalous behavior of the electron spectrum near the endpoint of tritium β-decay is offered. It is based on a new theory of mass in which mass becomes a dynamical variable, and the electron in the tritium β-decay has a narrow mass distribution. The predicted Kurie plots explain the main feature ("$m_\nu^2 < 0$") of this anomalous behavior.

Tritium beta decay shows anomalous behavior, observed in many experiments since 1991 [1–9]. Though various theoretical explanations have been offered [10–12], there is as yet no consensus on the solution of the puzzle. The anomaly consists of two effects. (a) The Kurie plot lies above a certain straight line (the theoretical result for a massless neutrino) near the endpoint, and overshoots that endpoint [13]. This is the effect that is fit by the infamous "negative neutrino mass squared" parameterization. (b) There is a narrow and low "bump" in the Kurie plot starting a few (5-10) eV below the endpoint [14].

We show that if the electrons emitted in tritium β-decay have a narrow mass distribution instead of the sharp mass m_0 assumed in the present day standard theory, the endpoint-overshoot effect (a) above, can be explained. This idea is based on a theory of mass developed by one of us; a few details will be given below. The electron antineutrino may be massless or have real mass $m_\nu > 0$, though the latter case gives a better fit, see later. The half-width Δm_e of the electron mass distribution must of course be much smaller than the central value m_0 ($\sim 1/2$ MeV, units: $\hbar = c = 1$), and the fits below suggest $\Delta m_e \approx m_\nu$.

It is easy to see qualitatively why such a mass distribution would imply the endpoint-overshoot effect. At a value of electron momentum p far from its endpoint value the distribution would produce a narrow scatter of events with electron energies both above and below $E_0(p) \equiv \sqrt{p^2 + m_0^2}$. But near enough to the *endpoint* $p = p_{max}$ (Definition: p_{max} is calculated from energy conservation for antineutrino momentum $q = 0$ and mass $m_\nu \geq 0$ on the standard theory, electron mass sharp $= m_0$) this scatter would become unsymmetrical because values of electron mass $m > m_0$ would be ruled out by energy conservation. At p_{max} the electron count goes to zero on the standard theory. But there would still be counts on the mass distribution theory due to values $m < m_0$. Correspondingly, the new Kurie plot $K(p)$ would coincide with the old one $K_0(p)$ for p far from the endpoint, but at $p = p_{max}$, where $K_0(p_{max}) = 0$, $K(p_{max})$ would be positive. Thus $K(p)$ would overshoot the endpoint by an amount of order Δm. Figure 1 illustrates this quantitatively and in detail.

To avoid some obvious misunderstandings we add a few words here on the theory [15,16] underlying this proposed solution. Some further details will be given in the concluding remarks.

The unsharp mass of the electron (the mass distribution) is *not* due to decay channels—the electron is stable. Rather, mass m of any elementary particle becomes a new dynamical variable alongside linear momentum $\mathbf{p}$ and

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energy E. Mass m is conjugate to a new length variable λ just as $\mathbf{p}$ and E are to position $\mathbf{r}$ and time t, and roughly obeys the uncertainty relation $\Delta m\,\Delta\lambda \geq 1/2$ in any state. The new dimension λ enters this theory as a microscopic length which modifies the usual point-particle causality of the 4-D theory at small distances [15]. It prevents the instantaneous action of source point on field point, and can be thought of as giving (massive) elementary particles a finite size $\sim \lambda$ [17]. The uncertainties Δm and $\Delta\lambda$ depend on the electron's state, here a free electron state. Thus Δm here is expected to be different from the Δm for, say, an electron bound in an atom or molecule.

This theory is invariant under the Poincaré group in particular, so that momentum and energy are conserved. Further, it implies the usual relation $E = \sqrt{p^2 + m^2}$ for free particles.

Traditionally in physics every particle had a definite (sharp) mass in the absence of interactions leading to decays. But both theory and experiment in the last fifteen years suggest that we may be in a transitional stage in our understanding of elementary particle mass. The concept of "particle mixing" has entered particle physics. What this implies is that there exist particles, some observable, which have no definite mass. (We mean here: apart from the mass widths implied by possible decay channels.) Examples are the three kinds of neutrinos, K^o and $\bar{K}^o$, the "primed" quarks d', s', and b', which directly enter the weak interaction Lagrangian, etc. These particles "oscillate": their states are superpositions of a few other particle states with sharp mass, the "mass (eigen)states". In other words, the oscillating particles have (simple) mass distributions.

However, there are inconsistencies and lacunae in this transitional theory of mass. For example: (i) Among elementary particles observable as free particles there are the heavy boson Z^o and the photon γ, which are mass eigenstates, and then there are the neutrinos ν_e, ν_μ and ν_τ, which are mixtures. (ii) Further, though the term "mass (eigen)state" has entered the literature [18], the corresponding "mass operator" for the general particle state has never been defined. (iii) There is no theory for the mixing angles, nor for the discrete and finite mass spectra we see in nature. (iv) Why are massless neutrinos necessarily only left-handed?

Suppose then that the electron has a narrow mass distribution $P(m)$. Whereas the usual free (Interaction Picture) quantum electron field appearing in the S-operator is [19]

$$\widehat{\psi}_0(x) = (2\pi)^{-3/2} \int d^3p\, (m_0/E_o)^{1/2}\, e^{ip\cdot x}\, u(\boldsymbol{p}, m_0)\, a(\boldsymbol{p}) + \text{antiparticle part},$$

the corresponding 4-D field with the mass distribution *amplitude* $A(m)$ is

$$\widehat{\psi}(x) = (2\pi)^{-3/2} \int \int dm\, d^3p\, (m/E)^{1/2}\, A(m)\, e^{ip\cdot x}\, u(\boldsymbol{p}, m)\, a(\boldsymbol{p}, m) + \text{antiparticle part}, \tag{1}$$

where

$$E(\boldsymbol{p}, m) \equiv \sqrt{\boldsymbol{p}^2 + m^2}, \quad (i\gamma \cdot p + m)u(\boldsymbol{p}, m) = 0, \quad [a(\boldsymbol{p}, m), a^\dagger(\boldsymbol{p}, m)]_+ = \delta(\boldsymbol{p} - \boldsymbol{p}')\delta(m - m'),$$

and where $P(m) \equiv |A(m)|^2$ and $\int_0^\infty dm\, |A(m)|^2 = 1$. Note that both $\widehat{\psi}_0(x)$ and $\widehat{\psi}(x)$ have dimension $L^{-3/2}$ ($L \equiv$ length) as they must to give a correctly normalized interaction Lagrangian.

Assume now that the electron field, Eq. (1), appears in the S-operator. Then on taking the S-matrix element corresponding to the decay $^3H \to {}^3He^+ + e + \bar{\nu}$, the electron differential spectrum $\mathcal{N}(p, m)$ (where, by definition, $\mathcal{N}(p, m) \equiv$ number of electrons per unit time with momentum p and mass m in the intervals $(p, p+dp)$ and $(m, m+dm)$, respectively) becomes [20]

$$\mathcal{N}(p, m) \equiv N(p, m)P(m), \quad N(p, m) \propto p^2 q E_\nu. \tag{2}$$

Here, $p \equiv |\boldsymbol{p}|$ is electron momentum, $q = |\boldsymbol{q}|$ is antineutrino momentum, $E_\nu \equiv \sqrt{q^2 + m_\nu^2}$ is antineutrino energy with a (real) antineutrino mass $m_\nu \geq 0$, and m is electron mass.

Energy conservation reads

$$Q \equiv M(^3H) - M(^3He^+) = E_e + E_\nu, \tag{3}$$

where (as usual) we neglect the $^3He^+$ recoil energy and for simplicity treat the case of a single $^3He^+$ final state. In the NR (nonrelativistic) case valid for tritium β-decay we can expand the electron energy as

$$E_e = [p^2 + (m_0 + x)^2]^{1/2} \approx m_0 + T + x, \quad x \equiv m - m_0, \quad T \equiv p^2/2m_0, \tag{4}$$

correct to terms linear in x (assume $P(m) \approx 0$ unless $|x| \ll m_0$). If Eq. (4) is used in Eq. (3), NR energy conservation reads

$$T_0 = T + x + E_\nu, \qquad T_0 \equiv Q - m_0. \tag{5}$$

We remark here that T_0 is usually called E_0 ($\approx 18{,}570$ eV) in the experimental literature. Note that T is not the kinetic energy of the electron of momentum p and mass m because m_0, not m, appears in the denominator. But it is convenient to use this T because then the full m-dependence of E_e to $O(x)$ appears in the single term $+x$ in the NR case. Further, T *is* the kinetic energy in the standard theory, and is what is used in the experimental papers (where it is usually called E).

After putting q and E_ν in terms of p and x, we get the electron differential spectrum in the form

$$\mathcal{N}(p, m) \propto 2m_0 \, T \, (\, T_0 - T - x \,) \, [\, (\, T_0 - T - x \,)^2 - m_\nu^2 \,]^{1/2} \, P(m).$$

It is more convenient to use a spectrum $\mathcal{N}'(T, m)$ for the number of electrons with kinetic energy T between T and $T + dT$, etc., so we multiply the above spectrum by $dp/dT = m_0/\sqrt{2m_0 T}$. Changing to the convenient energy variable $y = T_0 - T$, we arrive at

$$\mathcal{N}'(y, x) \propto m_0 \sqrt{2m_0 \, (T_0 - y)} \, (\, y - x \,) \, [\, (\, y - x \,)^2 - m_\nu^2 \,]^{1/2} \, P(x) \, dydx, \tag{6}$$

where we wrote $\mathcal{N}'(y, x) \equiv \mathcal{N}'(T, m)$ and $P(x) \equiv P(m)$ for notational simplicity. Note that since momentum p (or equivalently, kinetic energy $T = p^2/2m_0$) is measured in the experiments *but not mass* m, the differential spectrum of interest is

$$\mathcal{N}'(y) \equiv \int \mathcal{N}'(y, x) dx. \tag{7}$$

Integrated spectrum. In the experiments [1–9], when a Kurie plot is given, it is the integrated spectrum (differential spectrum integrated in T from some T to the endpoint). Hence we consider the integral of $\mathcal{N}'(y)$ from the endpoint $y = 0$ to some $y > 0$. The only values of y of interest are in the endpoint region $y << T_0$, so we can put $T_0 - y \to T_0$ in Eq. (6). Then from Eqs. (6) and (7):

$$\mathcal{N}^{int}(y) \propto m_0 \sqrt{2m_0 \, T_0} \int_{-n\Delta m}^{y - m_\nu} dx \, P(x) \int_{x + m_\nu}^{y} dy' (\, y' - x \,) \, [\, (\, y' - x \,)^2 - m_\nu^2 \,]^{1/2}. \tag{8}$$

The limits were determined from energy conservation, Eq. (5), on the assumption that the y'-integral is done first for a given value of x. In the x-integral, Δm is the half-width of the mass distribution, and n is some small integer (2 or 3) such that $P(x)$ is considered effectively zero for $x \le -n\Delta m$.

Remarkably, the y'-integral in Eq. (8) can be done analytically. Then we define a normalized [21] integrated Kurie plot $K(y)$ by dividing out some factors and taking the cube root of the resulting integral $I(y)$:

$$I(y) \equiv \int_{-n\Delta m}^{y - m_\nu} dx \, P(x) \, [\, (\, y - x \,)^2 - m_\nu^2 \,]^{3/2}, \quad m_\nu - n\Delta m < y << T_0, \tag{9a}$$

$$K(y) \equiv [I(y)]^{1/3}. \tag{9b}$$

The integral $I(y)$ must be done numerically, and we chose $P(x)$ to be the gaussian

$$P(x) = \frac{1}{\sqrt{2\pi} \, \sigma} \exp\left(-\frac{x^2}{2\sigma^2} \right) \tag{10}$$

for the numerical integration, in which case we set $\Delta m = \sigma$, the standard deviation of the gaussian. But see the concluding remarks.

There are two mass parameters available, σ and m_ν. It is convenient to non-dimensionalize Eqs. (9) and (10) by using one of them. We chose m_ν, and denoted the dimensionless quantities by carets:

$$\widehat{x} \equiv x/m_\nu, \quad \widehat{y} \equiv y/m_\nu, \quad \widehat{\sigma} \equiv \sigma/m_\nu, \quad \widehat{T}_0 = T_0/m_\nu; \qquad \widehat{P}(\widehat{x}) \equiv m_\nu P(x), \quad \widehat{I}(\widehat{y}) \equiv I(y)/m_\nu^3, \quad \widehat{K}(\widehat{y}) \equiv K(y)/m_\nu.$$

Then the quantity plotted in Fig. 1 is $\widehat{K}(\widehat{y}) \equiv [\widehat{I}(\widehat{y})]^{1/3}$, where

$$\widehat{I}(\widehat{y}) \equiv \int_{-n\widehat{\sigma}}^{\widehat{y} - 1} d\widehat{x} \, \widehat{P}(\widehat{x}) \, [\, (\widehat{y} - \widehat{x})^2 - 1 \,]^{3/2}, \qquad 1 - n\widehat{\sigma} < \widehat{y} << \widehat{T}_0, \tag{11}$$

3

for various values of the ratio $\widehat{\sigma} \equiv \sigma/m_\nu$.

Note that in the sharp mass limit $\widehat{P}(\widehat{x})$ goes to a delta function, $\widehat{I}(\widehat{y})$ becomes $(\widehat{y}^2 - 1)^{3/2}$, and

$$\widehat{K}(\widehat{y}) \rightarrow (\,\widehat{y}^2 \, - \, 1\,)^{1/2} \approx \widehat{y} \, - \, 1/2\widehat{y}, \quad \text{for} \quad \widehat{y} >> 1.$$

This is a straight line of slope $+1$ (in $\widehat{y}$) far from the endpoint, but near the endpoint it curves down and vanishes at $\widehat{y} = 1$ below the endpoint, the well-known result for a massive neutrino in the standard theory. (For $m_\nu = 0$, the corresponding plot of $K(y)$ would be a straight line all the way to $y = 0$.)

In the present, mass unsharp theory for the case $m_\nu = 0$ we must return to $K(y)$, Eq. (9b), with m_ν set to zero. This can be non-dimensionalized by dividing all quantities by suitable powers of σ. We have omitted this graph here; the Kurie plot lies approximately half way between the plotted curves for $\widehat{\sigma} = 1.0$ and 1.5 shown in the figure.
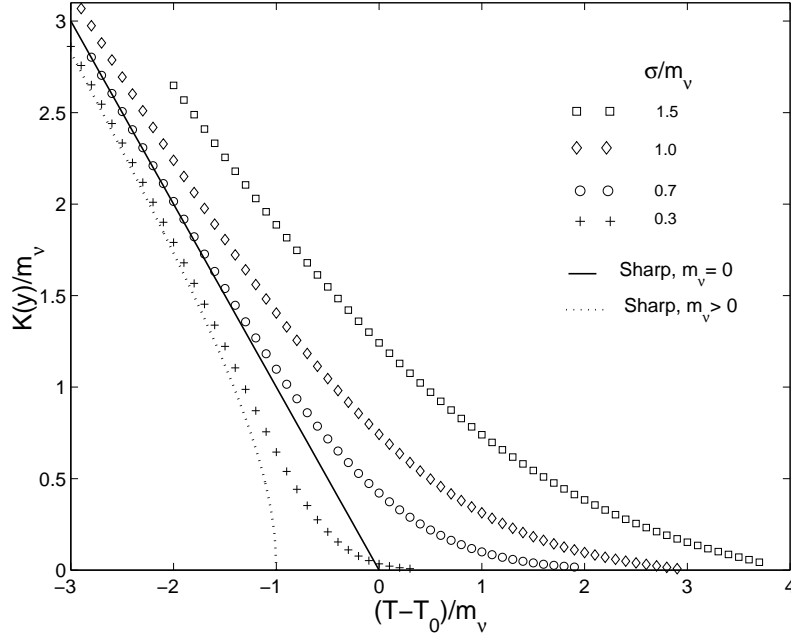

FIG. 1. The non-dimensionalized Kurie plots $\widehat{K}(\widehat{y}) \equiv K(y)/m_\nu$ against $-\widehat{y} \equiv (T - T_0)/m_\nu$ (the latter for ease of comparison with experimental results), for $\widehat{\sigma} = 0.3, 0.7, 1.0,$ and 1.5 from left to right. Also shown are the Kurie plots of $\widehat{K}_0(\widehat{y})$ in the sharp mass case for massive and massless neutrino.

Discussion of the results. Fig. 1 shows the non-dimensionalized Kurie plot for the cases $\widehat{\sigma} = 0.3, 0.7, 1.0,$ and 1.5. For ease of comparison with experimental results, however, we have plotted $-\widehat{y} \equiv (T - T_0)/m_\nu$ on the horizontal axis. Also shown are the Kurie plots in the sharp mass case: $\widehat{K}_0(\widehat{y}) = (\,\widehat{y}^2 \, - \, 1\,)^{1/2}$ for $m_\nu > 0$ and the straight line $\widehat{K}_0(\widehat{y}) = -\widehat{y}$ for $m_\nu = 0$ (the latter is the dimensional Kurie plot $K_0(y) = -y$ arbitrarily normalized with an $m_\nu \neq 0$). $K(y)$ stays above the straight line and overshoots the endpoint $y = 0$ ($T = T_0$) by about $2m_\nu, 3m_\nu,$ or $4m_\nu$ in the cases $\widehat{\sigma} = 0.7, 1.0,$ and 1.5, respectively. If one knew or guessed a value for m_ν, this would give the predicted overshoot quantitatively. The comparison with experiment would be clearer if the data at and just beyond the endpoint were cleaner, as previously mentioned [13]. The curves for the values $\widehat{\sigma} < 0.3$ lie below the straight line and do not overshoot the point $T = T_0$. There is no indication in any of these curves of effect (b), the "bump". The plots in the Figure resemble the experimental plots in Refs. [1,2,3,5,8, and 9] in a general way. In particular, the plot for $\widehat{\sigma} = 0.7$ appears to be an especially good fit to the experimental points in Fig. 1 of the recent work [9].

Concluding remarks. (a) The gaussian, Eq.(10) was chosen arbitrarily as a typical narrow mass distribution, but calculations using other mass distribution functions indicate that the Kurie plots do not depend sensitively on the shape of the distribution so long as it is narrow. We should mention that the underlying theory [15] is capable of predicting this mass distribution uniquely. Further, recent work [16] suggests how the effective 4-D field $\widehat{\psi}(x)$, Eq. (1), arises from the natural 5-D free quantum field $\widehat{\Psi}(x, \lambda)$ of this 5-D theory. However, these ideas are not yet in final form and anyway would be too lengthy to give here. (b) The possible objection that the uncertainty in the electron mass found in particle property tables is too small to explain this ^3H β-decay anomaly is no valid criticism of

this proposal. To repeat: if mass becomes a dynamical variable, its uncertainty depends on the state, and may vary wildly with that state. The value 0.3 ppm for the uncertainty quoted in the Tables [22] comes from the most precise atomic measurements. There is no reason for the mass width to be the same as that for a free electron especially if this "free" electron is acted on by a self-force. (A self-force was found essential in the classical theory to give a nonsingular self-interaction with all the correct properties [15] and in the quantum theory to enable the prediction of mass spectra for elementary particles [16]). To our knowledge no experiment measuring the mass width of a free electron, such as emerges in tritium β-decay, has ever been performed, though it would be simple in principle.(c) We can give an upper bound on $\sigma = \Delta m_e$ on the basis of our results as follows. We predict $\sigma = \widehat{\sigma} m_\nu \approx 0.7 m_\nu$ (see above). If $m_\nu < 1$ eV for example, then $\sigma < 0.7$ eV, or < 1.4 ppm.

ACKNOWLEDGMENTS

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[20] See, for example, D. H. Perkins, *Introduction to High Energy Physics*, 3rd ed. (Addison-Wesley, Reading, Mass., 1987), Sec. 7.2, for an elementary derivation of the spectrum $N(p, m)$.
[21] So normalized, our $K(y)$ has dimensions of M or L^{-1} ($\hbar = c = 1$), so cannot be directly compared in size to experimental Kurie plots, which usually display $(counts/s)^{1/3}$.
[22] First reference of Ref. [18], p. 65.